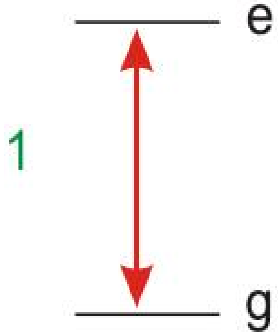
PHYS 514

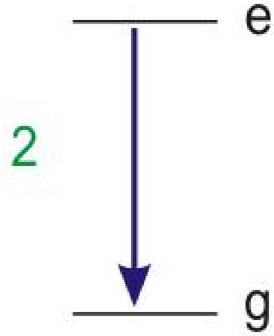
Light and atoms I - all classical

Note Title 2/28/2005

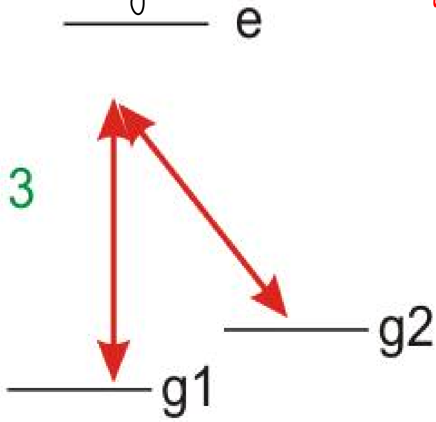
What happens in nature?



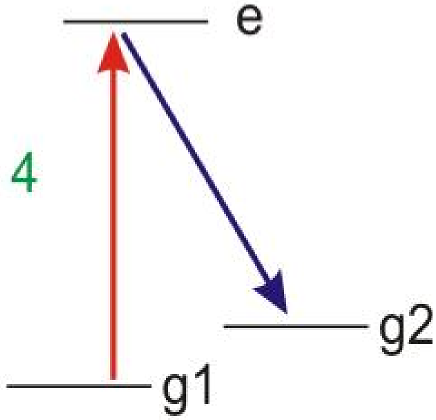
Rabi oscillations
(no decay) - classical light



spontaneous emission

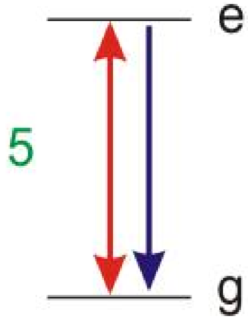


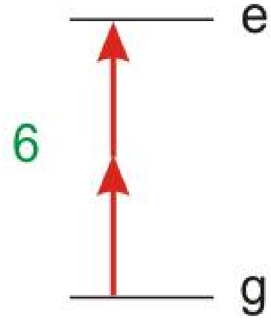
stimulated Raman transitions - classical light



spontaneous Raman scattering

resonance fluorescence

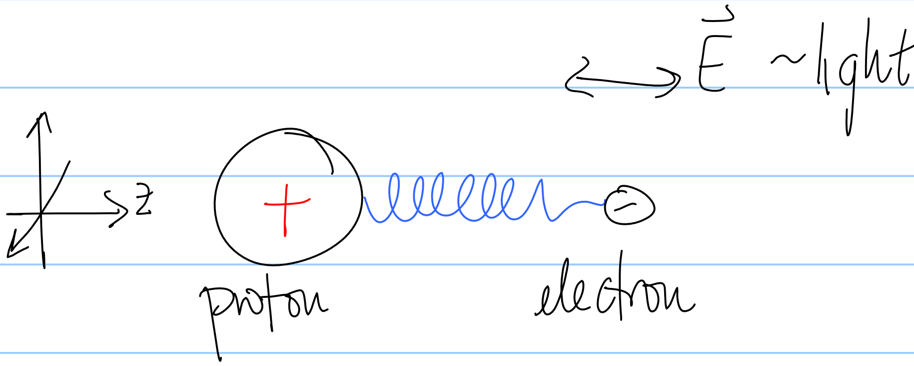




two-photon transitions

How do we describe these interactions?

How far can we get with a classical atom and classical light?



model of light interacting with an atom!

equation of motion: $\ddot{\vec{r}} + \omega_0^2 \vec{r} = -\frac{e}{m}\vec{E}$ forced harmonic oscillator

don't leave out the radiation reaction force!

The moving electron radiates! * see Corney section 8.1.1 and onwards

This has to be added in as a "friction" force (see Jackson, Corney)

radiating dipole: $\vec{p} = -ez\cos\omega t\,\hat{z} = p_0\cos\omega t\,\hat{z}$

$$\vec{E} = \frac{-p_0 k^2}{4\pi\varepsilon_0 r}\sin\theta\cos(kr-\omega t)\,\hat{\theta} \qquad k = \frac{2\pi}{\lambda}$$

$$\vec{B} = \frac{-1}{Z_0}\frac{p_0 k^2}{4\pi\varepsilon_0 r}\sin\theta\cos(kr-\omega t)\,\hat{\phi}$$

$Z_0 = \dfrac{1}{\varepsilon_0 c}$ impedence of free space $\simeq 377\,\Omega$

$\vec{S} = \vec{E}\times\vec{B}$ Poynting vector — radiated intensity

$\int d\Omega\,|\vec{S}|$ = radiated power = P

Recall Larmor formula for an accelerating charge:

$$P = \frac{e^2|a|^2}{6\pi\varepsilon_0 c^3}$$, where a is the acceleration

if this comes from a force F_r: $\displaystyle\int_0^{2\pi/\omega} F_r \dot{z}\,dt = \frac{-e^2}{6\pi\varepsilon_0 c^3}\int_0^{2\pi/\omega}(\ddot{z})^2\,dt$

$$\int_0^{2\pi/\omega} F_r \dot{z}\,dt = \frac{-e^2}{6\pi\varepsilon_0 c^3}\left\{\cancelto{0}{\dot{z}\ddot{z}\Big|_0^{2\pi/\omega}} - \int_0^{2\pi/\omega}\dot{z}\dddot{z}\,dt\right\}$$

so, identify: $\displaystyle F_r = \frac{e^2\dddot{z}}{6\pi\varepsilon_0 c^3}$

OK – take everything 1-dimensional along z

new equation of motion: $\ddot{z} + \omega_0^2 z = \frac{e^2}{6\pi\varepsilon_0 c^3 m}\dddot{z} - \frac{e}{m}E$

radiation reaction force is very small compared with binding force (something like 8 orders of magnitude)

so, $\ddot{z} \simeq -\omega_0^2 z$

$\dddot{z} \simeq -\omega_0^2 \dot{z}$

and rewrite our equation as: $\boxed{\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \frac{-e}{m}E_0 e^{i\omega t}}$

where $\gamma = \frac{e^2\omega_0^2}{6\pi\varepsilon_0 c^3 m}$

What happens if we just let the oscillator go ($E_0 = 0$)?

$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = 0$

for $\gamma \ll \omega_0$: damped oscillations:

$z = z_0 e^{-\gamma t/2} e^{-i\omega_0 t}$

I'm going to start using a notation where I have to take real parts.

which means that the "emitted" field is:

$$E(t) = E_0 e^{-i(\omega_0 - i\gamma/2)t} e^{ikr} \qquad (\text{for } t \geq 0)$$

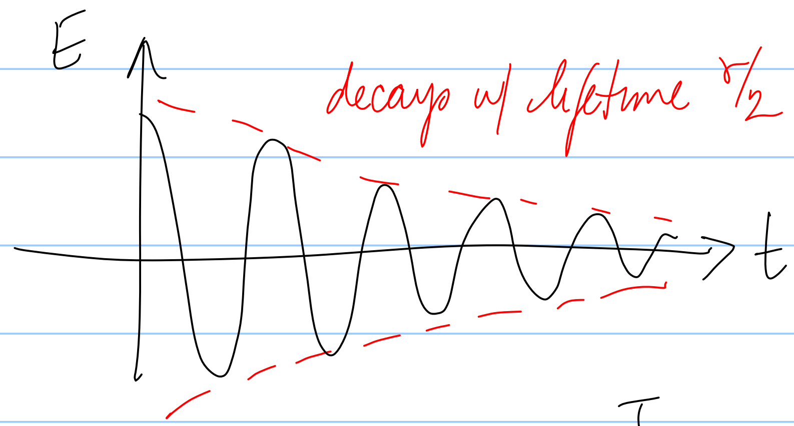
taking $\mathcal{F}[E(t)] = E(\omega) = \dfrac{-E(0)}{\sqrt{2\pi}} \dfrac{1}{i[(\omega-\omega_0)+i\gamma/2]}$ at one point in space

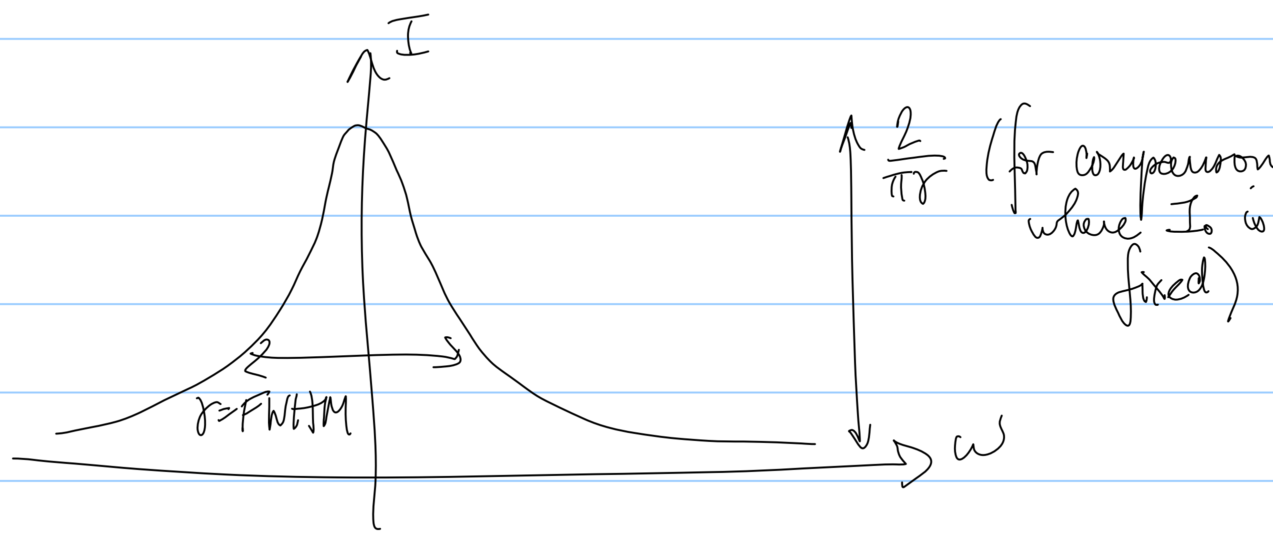
No longer monochromatic!

$$|E|^2 = I_0 \frac{\gamma/2\pi}{(\omega-\omega_0)^2 + \gamma^2/4}$$

Lorentzian distribution

$$I_0 = \int_0^\infty I(\omega)\, d\omega$$





Compare with ^{87}Rb: $\lambda = 780\,nm$

$\gamma_{classical} = 5.7$ MHz

real life: $\gamma = 5.98$ MHz!

$\boxed{\text{just luck!}}$ there are many things wrong with the results from this model

What about with driving field?

$$\ddot{z} + \gamma \dot{z} + \omega_0^2 z = \frac{-e}{m} E_0 e^{-i\omega t}$$

for a solution, try $z = z_0(\omega) e^{-i\omega t}$

this works $\longrightarrow$ $z_0 = \frac{-eE_0}{m} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$

What's the work done on the electron?

average power over one cycle:

$$\bar{P} = \frac{1}{T}\int_0^T dt \, \mathrm{Re}\left(-eE_0 e^{-i\omega t}\right) \mathrm{Re}(\dot{z}) = \int F v \, dt$$

$$= \frac{-1}{T}\int_0^T dt \, eE_0 \cos\omega t \left(i\omega z_0^* e^{i\omega t} - i\omega z_0 e^{-i\omega t}\right)$$

$$\vdots$$

$$= -eE_0 \omega \, \mathrm{Im}(z_0)$$

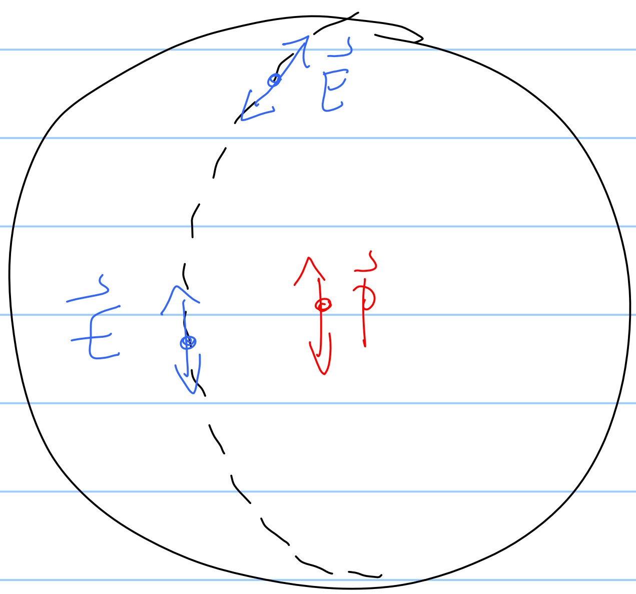
$$\bar{P} = \frac{(eE_0)^2}{2m} \frac{\gamma/2}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

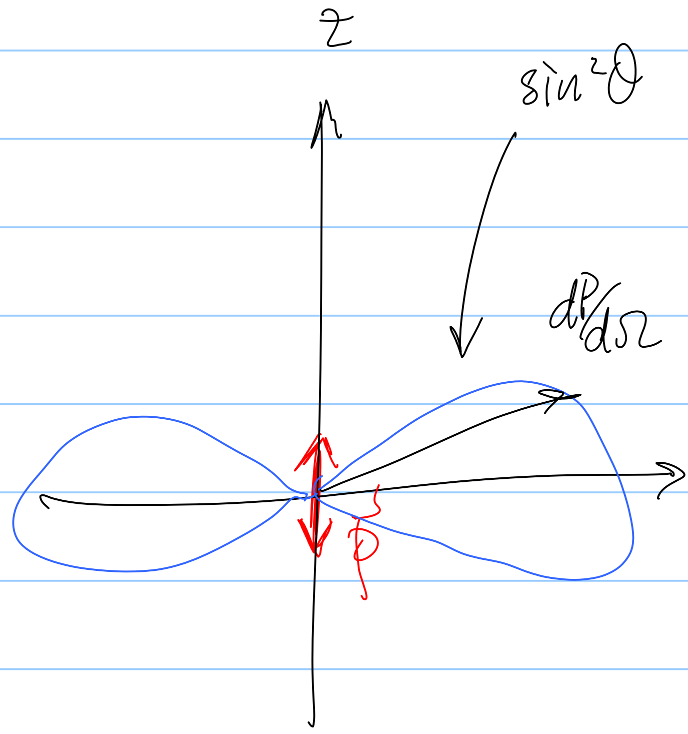
Our ability to drive has the same Lorentzian dependence!

What about the polarization of the emitted light?

in general, for a dipole $\vec{p}$: $\vec{E} = \frac{k^2}{4\pi\varepsilon_0}(\hat{r}\times\vec{p})\times\hat{r}\,\frac{e^{i(kr-\omega t)}}{r}$

linear oscillations





$$\frac{dP}{d\Omega} = \frac{1}{2Z_0}\left(\frac{k^2}{4\pi\varepsilon_0}\right)^2 \left|(\hat{r}\times\vec{p})\times\hat{r}\right|^2$$

circular oscillations

Classically, this happens with a magnetic field along $\hat{z}$ (see Foot, Chapter 1)

→ ignoring the radiation reaction force:

$$m\ddot{\vec{x}} = -m\omega_0^2\vec{x} - e\dot{\vec{x}} \times \vec{B}$$

no driving field, electron is harmonically bound (spherically)

$$\vec{B} = B\hat{z}$$

$$\ddot{\vec{x}} + 2\Omega_L \dot{\vec{x}} \times \hat{z} + \omega_0^2 \vec{x} = 0 \qquad \Omega_L = \frac{eB}{2m}$$

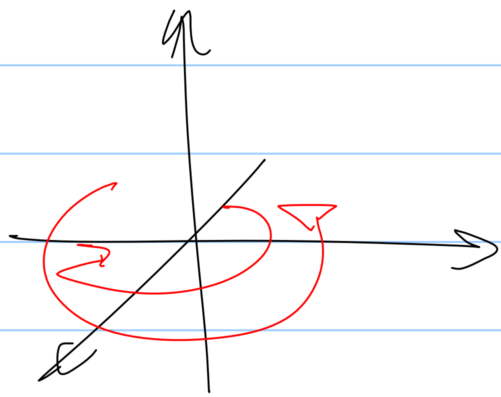
Larmor frequency

assume: $\Omega_L \ll \omega_0$

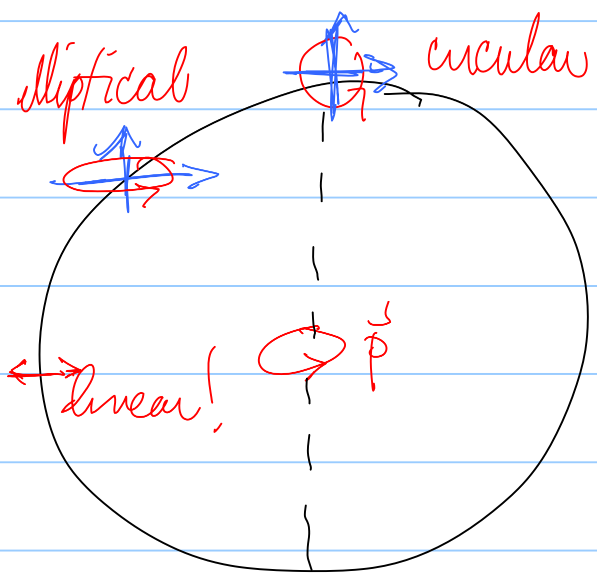
motion can be solved in the usual way for harmonic systems - there are 3 eigenfrequencies and eigenvectors

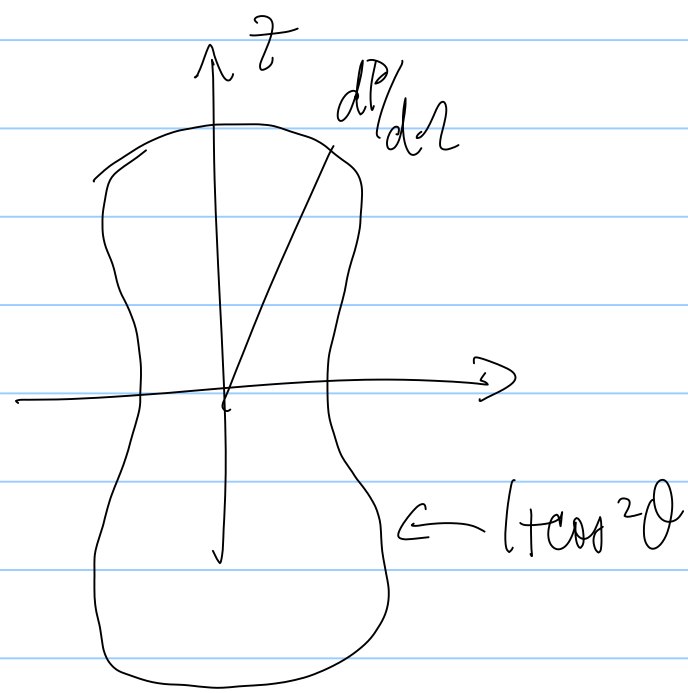
$$\omega = \omega_0,\ \omega_0 + \Omega_L,\ \omega_0 - \Omega_L$$

$$\vec{x} = \begin{pmatrix} 0 \\ 0 \\ \cos\omega_0 t \end{pmatrix},\ \begin{pmatrix} \cos(\omega_0+\Omega_L)t \\ \sin(\omega_0+\Omega_L)t \\ 0 \end{pmatrix},\ \begin{pmatrix} \cos(\omega_0-\Omega_L)t \\ -\sin(\omega_0-\Omega_L)t \\ 0 \end{pmatrix}$$



two circular orbits — CW and CCW





Corresponding quantum problem